

# Masses of $J^{PC} = 1^{-+}$ exotic quarkonia in a Bethe-Salpeter-equation approach

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We investigate the properties of mesons with the exotic  $J^{PC} = 1^{-+}$  quantum numbers. Starting out from the light-quark domain, where the  $\pi_1$  states are used as references, we predict the masses of analogous quarkonia for  $c\bar{c}$  and  $b\bar{b}$  configurations. We employ a covariant Dyson-Schwinger-Bethe-Salpeter-equation approach with a rainbow-ladder truncated model of quantum chromodynamics.

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## I. INTRODUCTION

After its origin several decades ago, hadron spectroscopy has seen increased interest in recent years. In particular, states not predicted or readily explained by the comprehensive quark-model calculations of the past, e. g., [1] have sparked both theoretical investigations as well as experimental searches at modern and future hadron-physics facilities; foremost, in the present manuscript we focus on mesons with exotic quantum numbers, which have been reviewed in great detail, e. g., very recently [2].

The approach we chose is the coupled Dyson-Schwinger-Bethe-Salpeter-equation (DSBSE) framework for the investigation of quantum chromodynamics (QCD), a well-established covariant variant [3–6] from a set of continuum quantum-field-theoretical methods [7–10] complementary to the well-known lattice-regularized QCD [11–13].

Concretely, our present work is built on top of recent studies of heavy quarkonia [14, 15], to which we add an immediate focus on the exotic meson quantum numbers  $J^{PC} = 1^{-+}$ , thus approaching the quarkonium spectrum from a slightly different perspective. In particular, the present study reaches out to the light-quark domain for anchoring in addition to the conventional quarkonium spectra in order to make use of experimental evidence presently at hand for the  $\pi_1$  states. Our setup employs the basic but symmetry-preserving rainbow-ladder (RL) truncation to study mesons by solving the quark Dyson-Schwinger equation (DSE) coupled to the meson  $q\bar{q}$  Bethe-Salpeter equation (BSE).

## II. SETUP AND MODEL

While we perform an RL-truncated study of Landau-gauge QCD in Euclidean space using the homogeneous BSE, we refer the reader to alternate routes of studies such as ours via a brief list of exemplary references to Coulomb-gauge QCD [16–19], the BSE in Minkowski space [20–22], an analogous but more general inhomogeneous vertex

BSE [23–25], and studies beyond RL truncation [26–30], from which the relevant literature can be grasped.

The strength of the setup chosen herein is its combination of computational and conceptional feasibility [31, 32] with the important features inherent to a systematic setup of a nonperturbative quantum-field-theoretical method. In particular, in QCD the implementation of chiral symmetry and its dynamical breaking as well as a realization of confinement and meson properties expected in the heavy-quark limit are paramount. Our study meets these requirements via properties demonstrated of RL truncated DSBSE studies with dressed quarks before, namely satisfaction of the axial-vector Ward-Takahashi identity [33–35], quark-confinement [36, 37], and adequate behavior towards the heavy-quark limit [38–40].

We use the homogeneous  $q\bar{q}$  BSE in RL truncation which reads

$$\Gamma(p; P) = -\frac{4}{3} \int_q^\Lambda \mathcal{G}((p-q)^2) D_{\mu\nu}^f(p-q) \gamma_\mu \chi(q; P) \gamma_\nu$$

$$\chi(q; P) = S(q_+) \Gamma(q; P) S(q_-), \quad (1)$$

where  $q$  and  $P$  are the quark-antiquark relative and total momenta, respectively, and the (anti)quark momenta are chosen as  $q_\pm = q \pm P/2$ . The renormalized dressed quark propagator  $S(p)$  is obtained from the corresponding DSE

$$S(p)^{-1} = (Z_2 i \gamma \cdot p + Z_4 m_q(\mu)) + \Sigma(p),$$

$$\Sigma(p) = \frac{4}{3} \int_q^\Lambda \mathcal{G}((p-q)^2) D_{\mu\nu}^f(p-q) \gamma_\mu S(q) \gamma_\nu. \quad (2)$$

$\Sigma$  is the quark self-energy,  $m_q$  is the current-quark mass chosen at a renormalization scale  $\mu$ ,  $D_{\mu\nu}^f$  represents the free gluon propagator and  $\gamma_\nu$  is the bare quark-gluon vertex's Dirac structure. Dirac and flavor indices are omitted for brevity.  $\int_q^\Lambda = \int^\Lambda d^4q / (2\pi)^4$  denotes a translationally invariant regularization of the integral, with the regularization scale  $\Lambda$  [41].

In a given truncation the effective interaction  $\mathcal{G}$  needs to be specified according to the aims of the study. We use the well-investigated and phenomenologically successful parameterization of Ref. [42], which reads

$$\frac{\mathcal{G}(s)}{s} = \frac{4\pi^2 D}{\omega^6} s e^{-s/\omega^2} + \frac{4\pi \gamma_m \pi \mathcal{F}(s)}{1/2 \ln[\tau + (1 + s/\Lambda_{\text{QCD}}^2)^2]}. \quad (3)$$

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The parameter  $\omega$  corresponds to an effective inverse range of the interaction, while  $D$  acts like an overall strength of the first term; they determine the intermediate-momentum part of the interaction, while the second term is relevant for large momenta and produces the correct one-loop perturbative QCD limit. We note that  $\mathcal{F}(s) = [1 - \exp(-s/[4m_t^2])]/s$  where  $m_t = 0.5$  GeV,  $\tau = e^2 - 1$ ,  $N_f = 4$ ,  $\Lambda_{\text{QCD}}^{N_f=4} = 0.234$  GeV, and  $\gamma_m = 12/(33 - 2N_f)$ , which is left unchanged from Ref. [42].

The parameters considered independent and free in our study are  $\omega$  and  $D$  (in addition to the current-quark masses), which we already used previously to achieve a surprisingly good phenomenological description of meson spectra [15, 43] in the heavy-quark domain. In the present study we refit the pair  $\omega$  and  $D$  using the same quark masses as in [15, 43] in order to specifically target the  $J^{PC} = 1^{-+}$  channel. Further details are specified below.

### III. EXOTIC QUANTUM NUMBERS

Exotic quantum numbers have been addressed within the DSBSE approach several times before [15, 38, 44–50], but have been in the actual focus of the work only in [45, 47].

In a short critique of Ref. [47] we would like to mention here in particular the claim that  $\omega$  and  $D$  are not independent so that “one can expect computed observables to be practically insensitive to  $\omega$  on the domain  $\omega \in [0.4, 0.6]$  GeV.” In our opinion, this is an unnecessary and over-restrictive assumption which actually prevents part of the possibilities a complete set of results may have. We note that there is a difference between having a one-parameter model at hand like in older studies to investigate effects the long-range behavior of the strong interaction might have on states of various kinds [38, 51–53] on one hand, or attempting a complete study of the possibilities within the parameter space of a given model interaction on the other hand. While in [47] the former is present and, in fact, a different value for the product of  $\omega D$  is used for excited and exotic states to attempt a description along the lines of beyond-RL premises, the latter is not, and so any conclusions drawn there are necessarily incomplete. For example, general statements about excited and exotic states are made in [47] based on light-quark results only, while heavy mesons are not considered, nor are those with axial-vector and tensor quantum numbers. In our present work, on the contrary, we show that axial-vector states may be the key to a proper understanding of, e.g., the exotic vector states in the light-meson spectrum.

In another short critique, namely of Ref. [45], we remark that, after an in-depth discussion of the DSBSE setup and its capabilities to describe states with exotic quantum numbers, the authors employ a strategy similar to ours. In particular they focus on states with  $J = 1$ , together with the obligatory pseudoscalar ground state, and obtain reasonable agreement with experimental data for vector, axial vector, and exotic vector ground states. Since the

choice of a separable kernel in the BSE limits the resulting equations and bound-state amplitudes, this set of results cannot easily be generalized or extended. For example, only two solutions are supported in the  $1^{-+}$  channel which also appear rather close together in mass, and an immediate comparison to the RL setup employed herein is difficult. At the time, possible interpretations as to why the results in [45] compared more favorably to the experimental situation than RL studies were mainly that there was lesser, and thus more appropriate, strength in the separable as compared to the RL kernel for light quarks.

With these two DSBSE studies of states with exotic quantum numbers in mind, we now very briefly sketch a few relevant issues before outlining our strategy towards meaningful statements in our approach.

In the DSBSE approach, states with exotic quantum numbers appear naturally in the setup of the quark-antiquark BSE. Owing to the ubiquity of the scalar product of the quark-antiquark relative and total momenta,  $q \cdot P$ , and its negative  $C$ -parity, the appearance of such states is a direct result of the dynamics and interaction considered. While in lattice QCD or constituent-quark models some explicit inclusion of gluonic degrees of freedom leads to the exotic value of  $C$ , a similar mechanism includes gluonic dynamics here *implicitly*; for more details and a thorough discussion, see [45]. For the present purpose and possible interpretations of our results, it is mainly important to remark here that our setup does not actually imply a non-hybrid interpretation.

Regarding the experimental situation of states seen in the  $1^{-+}$  channel, we are aware of an ongoing discussion about the nature and properties of the  $\pi_1$  states as they are listed in [54]. However, improvements are expected due to the wide efforts undertaken both experimentally and theoretically to make the overall quality and understanding of the hadron spectrum even better [55]. For the purposes of our argument, we assume enough credibility of experimental information on the  $\pi_1(1400)$  and  $\pi_1(1600)$  states to warrant our hypothesis, which is certainly the case. The  $\pi_1(2015)$  is neither needed for our fit setup, nor matched by any of our resulting states (our results are too high to match it) and so is omitted from our discussion.

### IV. STRATEGY

Naturally, a study using a truncated set of equations like ours has to be taken with a grain of salt. As a result, we employ the following strategy in pursuit of meaningful predictions for  $1^{-+}$  meson masses in the quarkonium spectrum: While an overall successful description of states with  $J \leq 2$  in RL truncation has been proven feasible in the heavy-quark domain [15], there is no reason *a priori* to assume similar success in the light-quark sector. On the contrary, a comprehensively successful phenomenological description of the same quality, even in the isovector case, would be a surprise. Nonetheless, subsets of observables

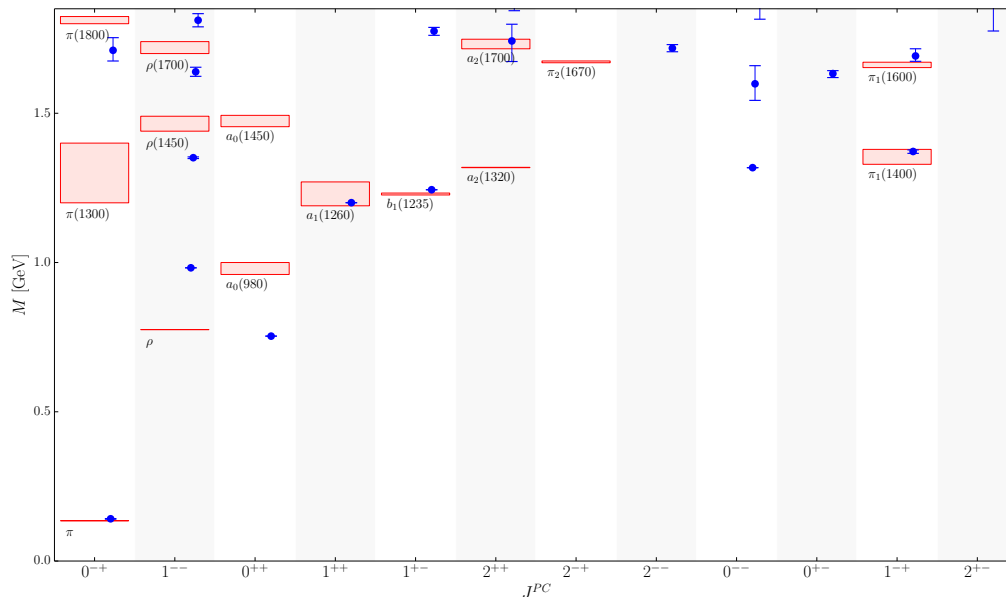


FIG. 1. Light isovector quarkonium spectra of mesons with  $J \leq 2$  including exotic quantum numbers, fitted to exotic-vector-related splittings. Calculated values (blue circles) are compared to experimental data (red boxes) from [54].

can be targeted by informed and suitably eclectic studies, which is our tactic herein as well.

In particular, as already indicated above, we make use of the experimental information available on the two lowest-lying exotic  $\pi_1$  states, the  $\pi_1(1400)$  and  $\pi_1(1600)$ , to test and constrain those meson mass splittings that may be appropriate, i. e., well-correlated to identify parameter sets potentially successful in the description of the  $1^{-+}$  channel also for the other quark masses. To find these, we use comparisons of calculated and experimental mass splittings and of sets of these on a grid  $\omega \in \{0.4, 0.5, 0.6, 0.7, 0.8\}$   $\text{GeV} \times D \in \{0.9, 1.1, 1.2, 1.3, 1.4, 1.5, 1.7\}$   $\text{GeV}^2$  in the isovector case. The pion mass is always fixed to its experimental value, which sets the current-quark mass in Eq. (2) to  $m_{u/d} = 0.003$   $\text{GeV}$  (given at a renormalization point  $\mu = 19$   $\text{GeV}$ ). We calculated meson masses for all  $J^{PC}$  quantum number sets for  $J \leq 2$  on the grid and investigated the corresponding  $\chi^2$  for a number of comparisons. A more detailed account of this strategy as well as an illustration is presented in [43].

To confirm that our region of interest is indeed well-captured by the grid defined above, we investigated the  $\pi_1$  states and found that a combination of the radial splitting between the ground and first excited  $1^{-+}$  states, together with the two splittings of each  $1^{-+}$  state to the ground-state pseudoscalar meson (pion) indeed provides a region of low  $\chi^2$  in the interior of our grid, which is the necessary prerequisite. We note that the best fit according to this configuration is given by  $\omega = 0.7$   $\text{GeV}$  and  $D = 1.4$   $\text{GeV}^2$ , for which the spectrum is plotted in Fig. 1. In all figures, our results are represented by blue circles, while experimental data are displayed as wide red boxes, where the height of the box indicates the size of the experimental error on the mass. Our error bars, where relevant, come

from extrapolated results in situations where propagator singularities prohibit a direct calculation; details on the source of this problem and our extrapolation strategy can be found in the appendices of [14, 15, 53, 59, 60].

The next step is to identify a set of non-exotic splittings that provide similar results for our fitting-attempts, i. e., a reasonable correlation to the exotic case. In search of such a set we have found that axial-vector states seem to be good indicators. This is not entirely surprising; for example, as mentioned above, Ref. [45] focused on states from the  $J = 1$  family and obtained a good overall description, including exotic states, using a separable kernel for the BSE, while both axialvector and exotic  $J = 1$  states had been difficult to describe otherwise. Another indication for the importance and connection to axialvector states is the hadronic decay of a  $\pi_1$  into axialvector mesons and pions [2, 61, 62], a feature which requires more work on our part and will be investigated in the future; still, it is important to note already here that a simultaneous good description of axialvector, exotic vector, and pion states is an excellent basis for a reliable subsequent calculation of their hadronic transition amplitude.

With this in mind, we studied several sets of splittings and found that the combination of the ground-state pseudoscalar to each of the axial-vector ground states together with the intra-axialvector splitting provide best guidance. The result is  $\omega = 0.7$   $\text{GeV}$  and  $D = 1.5$   $\text{GeV}^2$ . As indicated by the proximity of the best-fit parameter sets, the results are indeed very close to each other and we therefore use this configuration to attempt predictions of  $1^{-+}$  states for other quark masses.

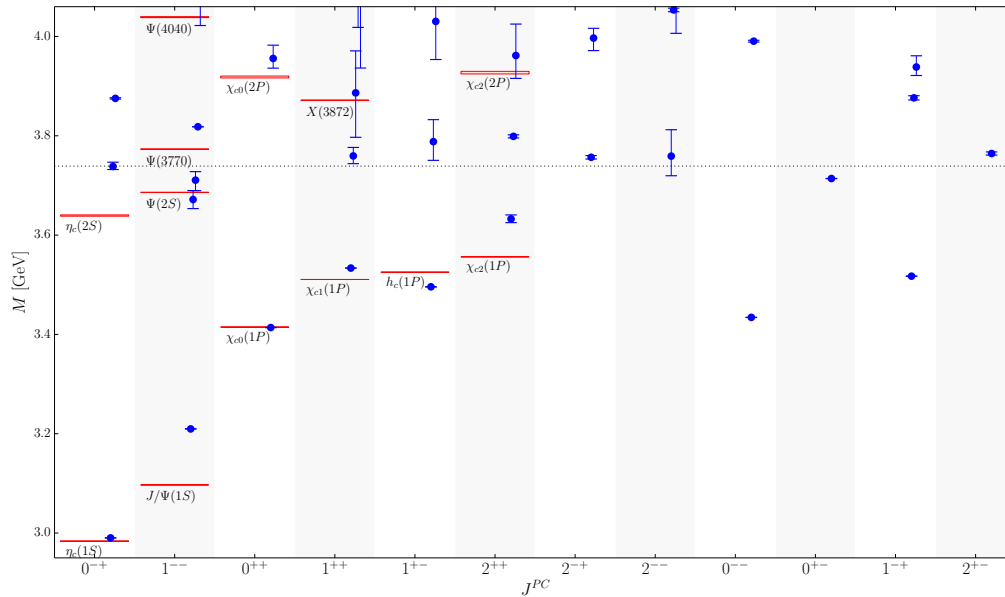


FIG. 2. Charmonium spectra, fitted to axial-vector-related splittings. Calculated values (blue circles) are compared to experimental data (red boxes) from [54, 56–58]. The horizontal dashed line marks the lowest open-flavor threshold.

## V. RESULTS AND DISCUSSION

First, we proceed to charmonium, where the same fitting procedure yields  $\omega = 0.6$  GeV and  $D = 0.9$  GeV<sup>2</sup>, for which the spectrum is plotted in Fig. 2. In addition to our results (blue circles) and experimental data (red boxes), the lowest open-flavor threshold is plotted as a horizontal dashed line. With the pseudoscalar and axial vector ground states well-met by the fit, we observe further adequate description of the scalar ground state. On the other hand, our calculation misses the vector and (pseudo-)tensor ground states, which is simply a consequence of the different anchoring and targets of the fit, as compared to our results in [15].

On a similar note, one observes that the level ordering of the ground states in the axialvector  $1^{++}$  and  $1^{+-}$ , while correct for the light-quark case, is reversed in charmonium. This is a result of a rather straight-forward dependence of this particular splitting on the model parameter  $D$ : small values of  $D$  result in the correct level ordering, while increasing  $D$  at some point flips the level ordering and increases the splitting in the wrong direction. In our study, the requirement to reproduce the intra-axial-vector ground-state splitting is combined with a second requirement, which together yield a good description of the exotic light meson spectrum, namely to reproduce the pseudoscalar-axialvector splittings. The value of  $D$  found via the fit to this combination thus determines the axial-vector level ordering and splitting and can miss it by a small amount. Apparently, the charmonium result for  $D = 0.9$  GeV<sup>2</sup> is beyond the flipping value with the splitting still small, albeit reversed, which is acceptable for our purposes.

The lowest charmonium state in the  $1^{-+}$  channel has a mass of about 3.52 GeV, the next comes out at about 3.88 GeV.

The situation is similar in the bottomonium case, where our fit arrives at  $\omega = 0.7$  GeV and  $D = 0.8$  GeV<sup>2</sup>, for which the spectrum is plotted in Fig. 3. While in this case our description of all measured ground states is rather accurate, our results for the excited states in many channels seem to be too low in general. Our resulting  $1^{-+}$  ground state appears at 9.82 GeV and the next at 10.02 GeV.

While we set out from the isovector  $\pi_1$  states, we now return to the light-quark domain for a brief comment on the isoscalar case. In our RL study, the BSE kernel does not account for flavor-mixing. While one can always mix the RL-BSE results afterwards [51], we refrain from doing so here, since there is no straight-forward and consistent way to implement such a mechanism for our present purposes. In a world with ideal  $SU(3)$  flavor mixing and assumed isospin-symmetry, an RL study provides the same results in the light-meson isovector and isoscalar channels. As a consequence, in such a world there should also be isoscalar versions of the  $\pi_1$  states, based on our simple setup. In addition, at a slightly higher mass, there would be analogous states with  $s\bar{s}$  quark content.

As far as the interpretation of all of these new states is concerned, they may be hybrids or mix with other types of states, much like in other approaches to this feature of QCD. If one makes use of a hybrid interpretation of our results, the immediate question arises, how one would find the partners in possible corresponding hybrid supermultiplets. However, a study with the ability to identify hybrid and conventional states among our results is beyond the scope of our present investigation. On

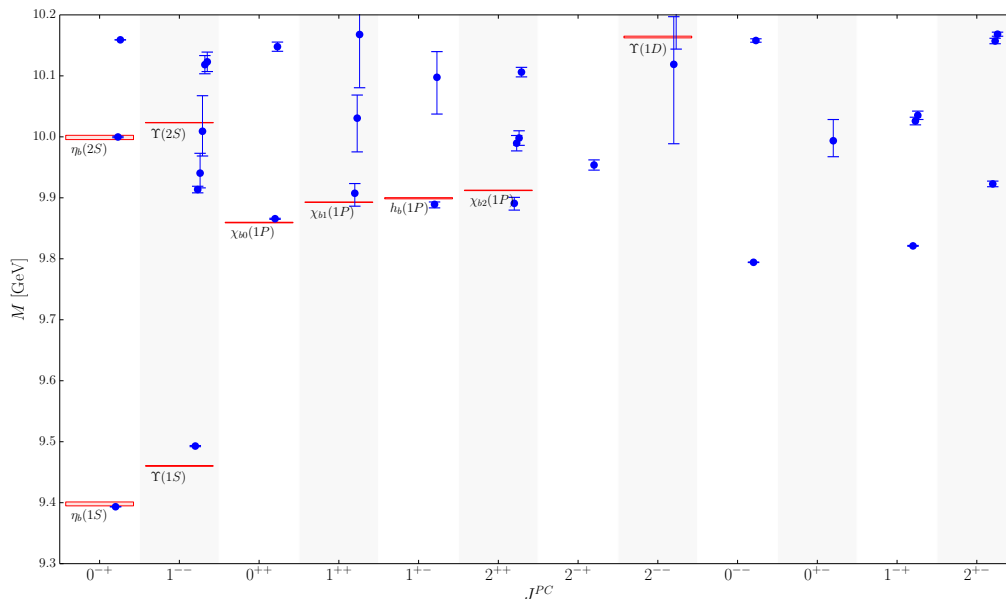


FIG. 3. Bottomonium spectra, fitted to axial-vector-related splittings. Calculated values (blue circles) are compared to experimental data (red boxes) from [54, 56]. The lowest open-flavor threshold is above all masses plotted in this figure.

another note, we would like to emphasize that our results for the  $1^{-+}$  channel, if compared to the results for hybrids presently available in the literature [2], are at the lower end of the apparent mass range: the predicted hybrid  $1^{-+}$  masses from lattice QCD, e.g., are substantially higher [12], whereas our results are in good agreement with, e.g., those from QCD sum rules [63].

While we have not aimed to describe states with the exotic quantum numbers  $J^{PC} = 0^{-+}, 0^{+-},$  or  $2^{+-}$ , we do obtain results in these channels and we have included our results for these states in our figures for the sake of completeness. Considering the strategy in our setup specifically anchored to the  $J = 1$  channels, we have no immediate reason to expect that our results present reasonable predictions. It is well conceivable that for the various exotic sets of quantum numbers there are different mechanisms at work and different parts of the interaction responsible for a good description. Naturally, one can turn to other approaches for guidance, which we may explore elsewhere. At present, our focus remains on a straight-forward study anchored to experimental data.

A final word of discussion concerns the fact that the states in our setup are bound states and not resonances, since a decay mechanism is not incorporated in the BSE kernel in RL truncation. In particular above the relevant decay thresholds, this feature must be met with caution. In a first step, a semi-perturbative way to calculate hadronic decay widths is available and has been tested successfully in the past, see [64] and references therein. Since, in particular, the  $\pi_1$  states are resonances, it remains to be seen whether such a treatment, using our current results, provides also hadronic widths of the correct size. Furthermore, considerable effects may also

be expected from final-state interactions [65].

## VI. CONCLUSIONS AND OUTLOOK

We presented a covariant study of the masses of prospective  $J^{PC} = 1^{-+}$  exotic mesons in both heavy and light quarkonia based on the DSBSE approach to QCD. Starting from a good description of the two lowest-lying  $\pi_1$  states and identifying a good “marker”, namely the axial vector channels, we transfer our assumptions to the other quarkonium systems and provide predictions for the masses of analogous states there. We see the lowest-lying  $1^{-+}$  state in both charmonium and bottomonium appear at about the same mass as the axial vector ground states. In summary, we have provided the hypothesis that, if our model assumptions are sufficient to capture the necessary physics and if the two lowest-lying  $\pi_1$  states are well-represented by our setup, then one should expect to find analogous states also in the other quarkonium systems as described in detail above.

To broaden our set of results we will investigate the decay properties of such states in the near future along the lines of previous studies of electromagnetic, leptonic, and hadronic transitions in our approach, which have been proven feasible and successful phenomenologically, e.g., in [52, 64, 66].

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